Indian Statistical Institute, Bangalore
B. Math (Hons.) Third Year
Second Semester - Analysis IV

Back Paper Exam — Date: 09th June 2022
Maximum marks: 100 — Duration: 3 hours

Section I: Each question carries 10 marks

1. Let X be a compact metric space. If $\mathcal{A}$ is a self-adjoint subalgebra of $C(X)$ that separtes points of X and nowhere vanishes, then prove that $\mathcal{A}$ is dense in $C(X)$.
2. State and prove Arzela-Ascoli Theorem.
3. Show that the set of all polynomials of degree at most 3 with coefficients from $[-1, 1]$ is compact in $C[0, 1]$.
4. Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a continously differentiable function such that $f'(x)$ is invertiable for all $x \in \mathbb{R}^n$. Prove that f is an open map.
5. Prove (the three identities in) Parseval's Theorem.
6. Suppose f is 2π-periodic differentiable function with $f' \in \mathcal{R}[-\pi, \pi]$. Prove that $s_n(f) \to f$.
7. Prove that $(\pi - |x|)^2 = \pi^2/3 - 4\sum_{n=1}^{\infty} \frac{cosnx}{n^2}$ for all $x \in [-\pi, \pi]$.
8. Prove $\sum_{n=1}^{\infty} \frac{sinnt}{n} = \frac{\pi - t}{2}$ for $t \in (0, \pi)$.
9. Prove that $x^2 = 4/3\pi^2 + 4\sum_{n=1}^{\infty}[\frac{cosnx}{n} - \pi\frac{sinnx}{n}]$ for $0 < x < 2\pi$.
10. Find total variation of $f(x) = x^3 - 2x^2 + x + 2$ and $g(x) = x^3 - 4x + 5$ on $[0, 1]$.